An Optimization Model for Reactive Power and Voltage Controls of Super High Voltage Grid Systems Using Pseudo-Inverse Method

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Abstract: - This paper develops an optimization model to control the excessive MVAR generation by super high voltage grid systems for maintaining the nodal voltages within the required acceptable margin. The model enhances an approximate solution on the condition of balanced real power. The Pseudo-Inverse Method of optimization is implemented as an optimal means of solving non square systems of equations based on Lagrange's Theory of optimization. It recommends the locations and ratings of the minimum required reactors from the preinstalled ones that to be in service for optimum nodal reactive power and voltage controls The model had been tested on the Iraqi Super High Voltage Grid System (400kV), and it proved to be efficient in the obtained results and reliable in convergence.

Index Terms:- Optimization Models, Pseudo-Inverse Method, High Voltage, Reactive Power Control, Voltage Control.

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1 Introduction

Present power flow optimization methods can be classified into, exact and approximate methods. Exact methods (1,2,3) take into account both real and reactive flows in obtaining the solution. While approximate methods (4,5) achieve simplified representations and possibly computational efficiencies by ignoring either the real or reactive equations. Approximate models are normally tailored for particular applications and do not have the generality inherent in the exact models.

The reduced gradient method of Dommel and Tinny (2), the Fletcher Powell method as developed by Sasson for power flow applications **(3),** and Carpentier's method based on satisfying the Khun-Tucker conditions (1), are all accurate and widely applicable.

While some have been developed to become computationally very efficient, a number of approximate models dealing exclusively with either the real or reactive power equations have also been developed. Hano, et Al (4) and Kumai, et Al **(5),** used primarily the reactive equations to develop a method for real time control of voltages and reactive powers. The need to adjust voltage magnitudes and reactive powers at times so that the overall solution is operationally implementable, led to the consideration of reactive equations.

2 Mathematical Formulation

Figure (1) is simulating a super high voltage grid system by representing the nodes into four categories from the point of view of their voltages [either they are within (m) or out (o) of the acceptable margin] and VAR absorption installations [either there are (r) or none (w)].
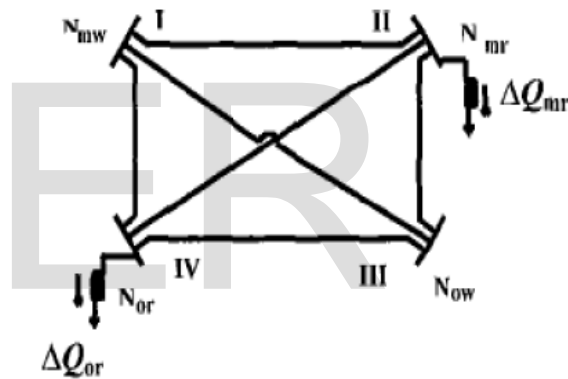

Figure (1) Schematic Single Line Diagram of the Optimization Mode

Considering the system in steady-state condition and the real power mismatch **ΔP** is zero, then the following linearized equations describe the effect of Q on V for the four categories of the system:-

$$\frac{dQ_{mw}}{dV_{mw}}\Delta V_{mw} + \frac{dQ_{mw}}{dV_{mr}}\Delta V_{mr} + \frac{dQ_{mw}}{dV_{ow}}\Delta V_{ow} + \frac{dQ_{mw}}{dV_{or}}\Delta V_{or} = 0 \ldots\ldots(1.a)$$

$$\frac{dQ_{mr}}{dV_{mw}}\Delta V_{mw} + \frac{dQ_{mr}}{dV_{mr}}\Delta V_{mr} + \frac{dQ_{mr}}{dV_{ow}}\Delta V_{ow} + \frac{dQ_{mr}}{dV_{or}}\Delta V_{or} = \Delta Q_{mr} \ldots\ldots(1.b)$$

$$\frac{dQ_{ow}}{dV_{mw}}\Delta V_{mw} + \frac{dQ_{ow}}{dV_{mr}}\Delta V_{mr} + \frac{dQ_{ow}}{dV_{ow}}\Delta V_{ow} + \frac{dQ_{ow}}{dV_{or}}\Delta V_{or} = 0 \ldots\ldots(1.c)$$

$$\frac{dQ_{or}}{dV_{mw}}\Delta V_{mw} + \frac{dQ_{or}}{dV_{mr}}\Delta V_{mr} + \frac{dQ_{or}}{dV_{ow}}\Delta V_{ow} + \frac{dQ_{or}}{dV_{or}}\Delta V_{or} = \Delta Q_{or} \ldots\ldots(1.d)$$

System (1) above can be rewritten as below:-

$$\begin{bmatrix} dQ_{mw}/dV_{mw} & dQ_{mw}/dV_{mr} & 0 & 0 \\ dQ_{mr}/dV_{mw} & dQ_{mr}/dV_{mr} & -E & 0 \\ dQ_{ow}/dV_{mw} & dQ_{ow}/dV_{mr} & 0 & 0 \\ dQ_{or}/dV_{mw} & dQ_{or}/dV_{mr} & 0 & -F \end{bmatrix} \begin{bmatrix} \Delta V_{mw} \\ \Delta V_{mr} \\ \Delta Q_{mr} \\ \Delta Q_{or} \end{bmatrix} = \begin{bmatrix} b1 \\ b2 \\ b3 \\ b4 \end{bmatrix} \quad ...(2)$$

Where E and F are unit diagonal matrices. Reducing System (2) using Newton-Gauss technique, it will be:-

$$\begin{bmatrix} A & 0 \\ B & -F \end{bmatrix} \begin{bmatrix} \Delta Q_{mr} \\ \Delta Q_{or} \end{bmatrix} = \begin{bmatrix} C1 \\ C2 \end{bmatrix} .....(3)$$

Or $D \ \Delta Q_R = C$........................ **(4)**

Where $D = \begin{bmatrix} A & 0 \\ B & -F \end{bmatrix}, \Delta Q_R = \begin{bmatrix} \Delta Q_{mr} \\ \Delta Q_{or} \end{bmatrix}, and, C = \begin{bmatrix} C1 \\ C2 \end{bmatrix}$

The ranks of the main and sub matrices are:-

Main Matrix	
D	$N_o \times N_r$
ΔQ_R	$N_r \times 1$
C	$N_o \times 1$

Where $N_r = N_{mr} + N_{or}, N_o = N_{or} + N_{ow}$

Sub Matrices			
A	$N_{0w} \times N_{mr}$	F	$N_{or} \times N_{or}$
B	$N_{or} \times N_{mr}$	$C1$	$N_{ow} \times 1$
E	$N_{mr} \times N_{mr}$	$C2$	$N_{or} \times 1$
ΔQ_{or}	$N_{or} \times 1$	ΔQ_{mr}	$N_{mr} \times 1$

The objective function of this model is:-

$$[\Delta Q_R]^T [\Delta Q_R] = Minimum.....(5)$$

To approach **ΔQ_R** optimum, the following algorithms will be followed during the iterative computational process:-

Algorithm I: - if **No** = Nr then:-
$$\Delta Q_R = D^{-1}C.....(6)$$

Algorithm II: - if No < Nr then:-
$$\Delta Q_R = D_1^+ C.....(7)$$
Where $D_1^+ = D^T(DD^T)^{-1}.....(8)$

Algorithm III: - if No > Nr then:-
$$\Delta Q_R \simeq D_2^+ C.....(9)$$
Where $D_2^+ = (D^TD)^{-1}D^T.....(10)$

D_1^+ and D_2^+ are the Pseudo-Inverses of the rectangular matrix **D(6,7)**, and their mathematical derivations are presented **in** Appendix-I.

3 Model's Flow Chart

The flow chart of the optimization model is shown in Figure (2). The dotted block represents the part where **ΔQ_R** is computed.
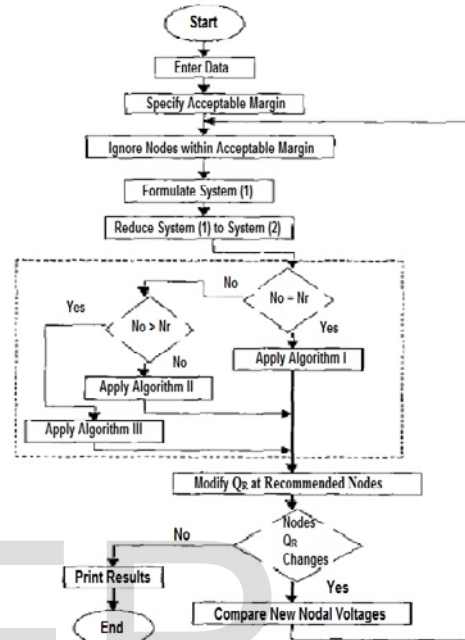

Figure -2- Optimization Model Flow Chart

4 Model's Performance

This optimization model had been tested successfully on the Iraqi Super High Voltage Grid System shown in Figure **(3)** at Minimum Load Condition of the year **2013.**
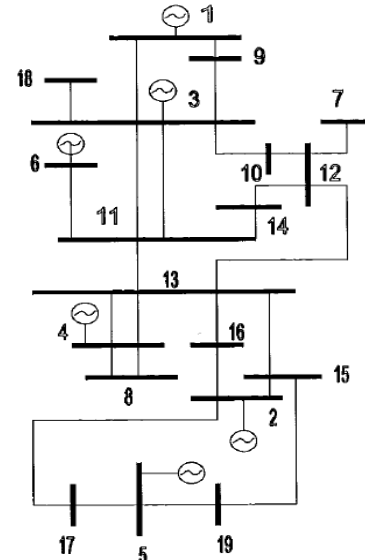

Figure-3- Iraqi Super High Voltage Grid System

To recognize the significance of the optimization model, three modes for MVAR control were applied on the system at the same load condition as follows:-

Mode I: - In this mode, No MVAR absorption assistance was made to the system ($\Delta Q_R = 0$).

Mode II:-All MVAR absorption resources were linked to the system (ΔQ_R =Maximum).

Mode III: - In this, the optimization model was tested on the system to achieve the optimum performance of the MVAR absorption resources by recommending the locations and ratings of the minimum required reactors from the preinstalled ones.

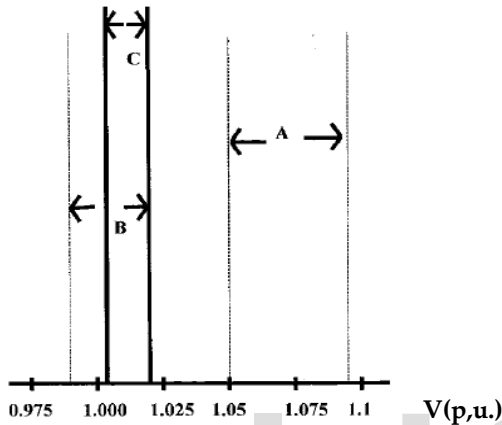Figure (4) translates nodal voltages in (p.u.) of the system into three margins.

Figure 4- Nodal Voltages Margins for the three modes of MVAR Control at Minimum Load Condition of the Iraqi Super High voltage Grid System

Margin A: Nodal Voltages Margin at (Mode 1).
Margin B: Nodal Voltages Margin at (Mode 11).
Margin C: Nodal Voltages Margin at (Mode 111).

While figure (5) translates reactive power generation / absorption required by the power stations for the three modes at minimum load condition of the system.
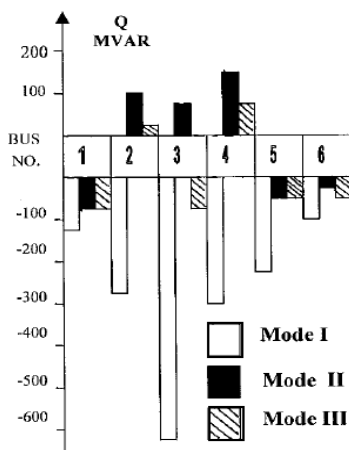

Figure -5- Reactive Power Generation/Absorption by the power stations for the three modes of MVAR control.

5 Conclusions

The proposed optimization model for the reactive and nodal voltage controls is based on linearized load flow reactive power equations and Lagrange's Theory of

optimization. The Pseudo-Inverse method of solving rectangular matrices is implemented via the algorithms of computing $[\Delta Q_R]$, when this matrix is not square.

The model proved to be reliable in convergence, the solution is obtained in few iterations and it needs less memory requirements due to that the Jacobian matrix is formulated with !4 of the linearized load flow equations (reactive power equations only) in the first iteration of the solution and it is reduced when ignoring the healthy nodes iteratively.

Three modes of MVAR control are implemented on the Iraqi National Super High Voltage Grid System at its worst operating condition, to recognize the significance and the efficiency of the proposed optimization model concerning the nodal voltage and MVAR controls. It has given better nodal voltage regulation at 25% of savings of total reactors ratings and has improved the performance of the generating stations from the point of view of reactive power generation/absorption, and then it improves the stability of the system.

Appendix- I

Having the system of equations [A][X]= [b], Where [A]= {a_{ij}} , i = l. ...m and j = l.. ..n. Its method of solution is dependent on whether [A] is square or not, and it can be obtained by one the following three algorithms:-

Algorithm I :-

If [A] is a square matrix, (m = n), then :-

[X]= [A]$^{-1}$[b]..,……………….. (I.1)

Algorithm II :-

If m<n, an optimal solution is achieved when:-

$$\sum_{i=1}^{n} X_i^{\,2} = [X]^T[X] = Minimum.....(I.2)$$

Optimum [X], is approached by Lagrange's theory:-

$$L = [X]^T[X] + [\lambda]^T[[A][X] - [b]]....(I.3)$$

$$\frac{\partial L}{\partial \lambda} = [A][X] - [b] = 0.....(I.4)$$

$$\frac{\partial L}{\partial X} = 2[X] + [A]^T[\lambda] = 0.....(I.5)$$

Equation (1.5) gives:-

$$[X] = -1/2[A]^T[\lambda]....(I.6)$$

Substituting for [X] in equation (1.4), results:-

$$[b] = -1/2[A][A]^T[\lambda]$$
$$[\lambda] = -2[[A][A]^T]^{-1}[b]....(I.7)$$

Substituting for [λ] in equation (l,6), gives: -

$$[X] = [A]^T[[A][A]^T]^{-1}[b] = [A_1]^+[b] \ldots (I.8)$$
$$\text{Where:- } [A_1]^+ = [A]^T[[A][A]^T]^{-1} \ldots (I.9)$$

And [A₁]⁺ is the Pseudo-Inverse of the rectangular matrix [A], when m<n.

Algorithm III :-

If m >n , an optimal solution is achieved when :-

$$\phi(X) = [[A][X] - [b]]^T[[A][X] - [b]] = \text{Minimum}$$
$$= [X]^T[A]^T[A][X] - 2[X]^T[A]^T[b] + [b]^T[b] \ldots (I.10)$$

Minimum **φ(x)** is approached when;-

$$\partial\phi/\partial X = 2[A]^T[A][X] - 2[A]^T[b] = 0$$
$$\text{then} \quad [X] = [[A]^T[A]]^{-1}[A]^T[b] \ldots (I.11)$$
$$= [A_2]^+[b]$$
$$\text{where:- } [A_2]^+ = [[A]^T[A]]^{-1}[A]^T \ldots (I.12)$$

And [A₂]′ is the Pseudo-Inverse of the rectangular matrix [A], when m>n.

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